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PERFORMANCE ANALYSIS FOR SUMMER AIR CONDITIONER WITH PRIORITY REPAIRS

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KEYWORDS

Parallel redundancy, Markovian process, supplementary variables, steady-state behavior, availability, profit function.

Abstract

In this study, the authors have considered a summer air conditioning system for a place in hot and dry weather. The authors have computed the availability and profit function for this system. Since the system under consideration is of Non-Markovian nature, the author has been used supplementary variables to convert this into Markovian. Probability considerations and limiting procedure have been used for mathematical formulations of the system. This mathematical model has been solved by using Laplace transform, to obtain probabilities of various transition states depicted in fig-1(b). Reliability, availability and M.T.T.F. for considered system have been obtained. Graphical illustration followed by a numerical computation has also been appended at last to highlight important results of this study.

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1. INTRODUCTION

Generally the systems under considerations are used for hot and dry outdoor conditions like Nagpur, Delhi, Bhopal and other place. The comfort conditions required in an air-conditioned space are 24^0C DBT (dry bulb temperature) and 60% RH (relative humidity). The arrangement of equipments required for an ordinary system has been shown in fig-1(a). The whole system is divided into five subsystems namely A, B, C, D and E. These subsystems are air dampers, air filter, cooling coils, adiabatic humidifier and water eliminator, respectively. All these subsystems are connected in series. The whole system reaches to failed state on failure of any of its subsystems A, C, D and E. On the other hand, the whole system works in reduced efficiency on failure of subsystems B. The whole system can also be failed due to wear-out reasons. Transition-state diagram for considered system has been shown in fig-1(b).

All failures follow exponential time distribution whereas all repairs follow general time distribution. Pre-emptive repeat policy has been adopted for repair purpose. Asymptotic behavior and a particular case, when repairs follow exponential time distribution, have been computed to enhance practical utility of the system.

2. ASSUMPTIONS

The following assumptions have been taken care throughout:

i) Initially, the whole system is operable.
ii) All failures follow exponential time distribution and are S-independent.
iii) Repairs follow general time distribution and are perfect.
iv) Repair facilities are always available.
v) The whole system can fail either due to failure of any of its constituent subsystem or due to wear - out.
vi) On failure of subsystem *B*, the whole system works in reduced efficiency state.
vii) Pre-emptive repeat policy has been adopted for repair purpose.

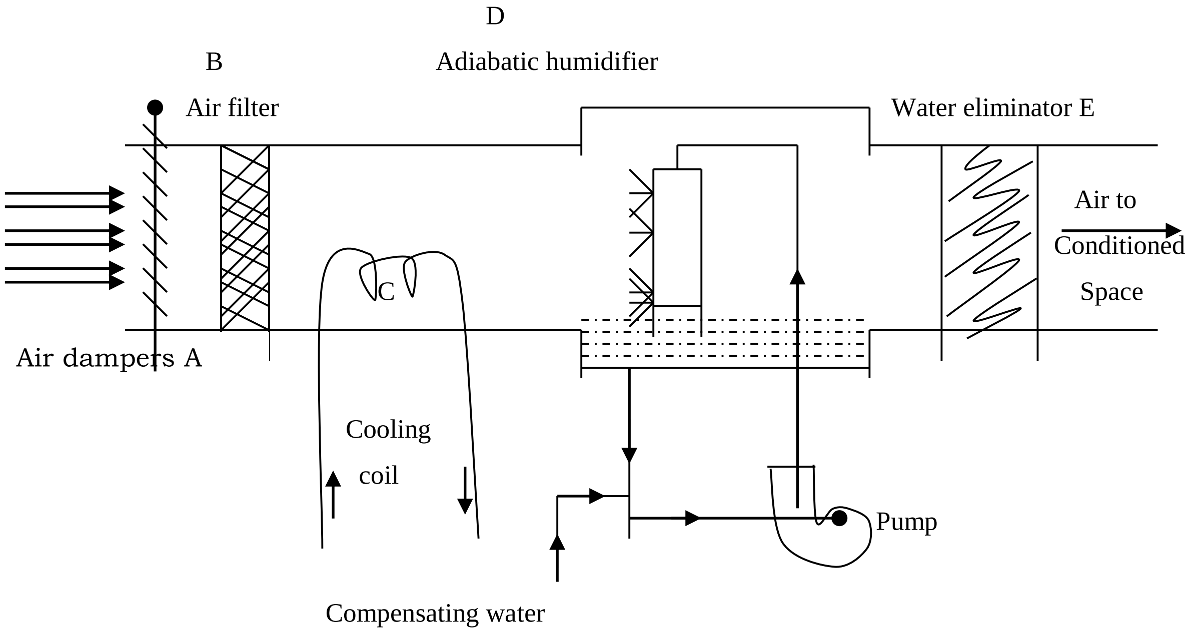
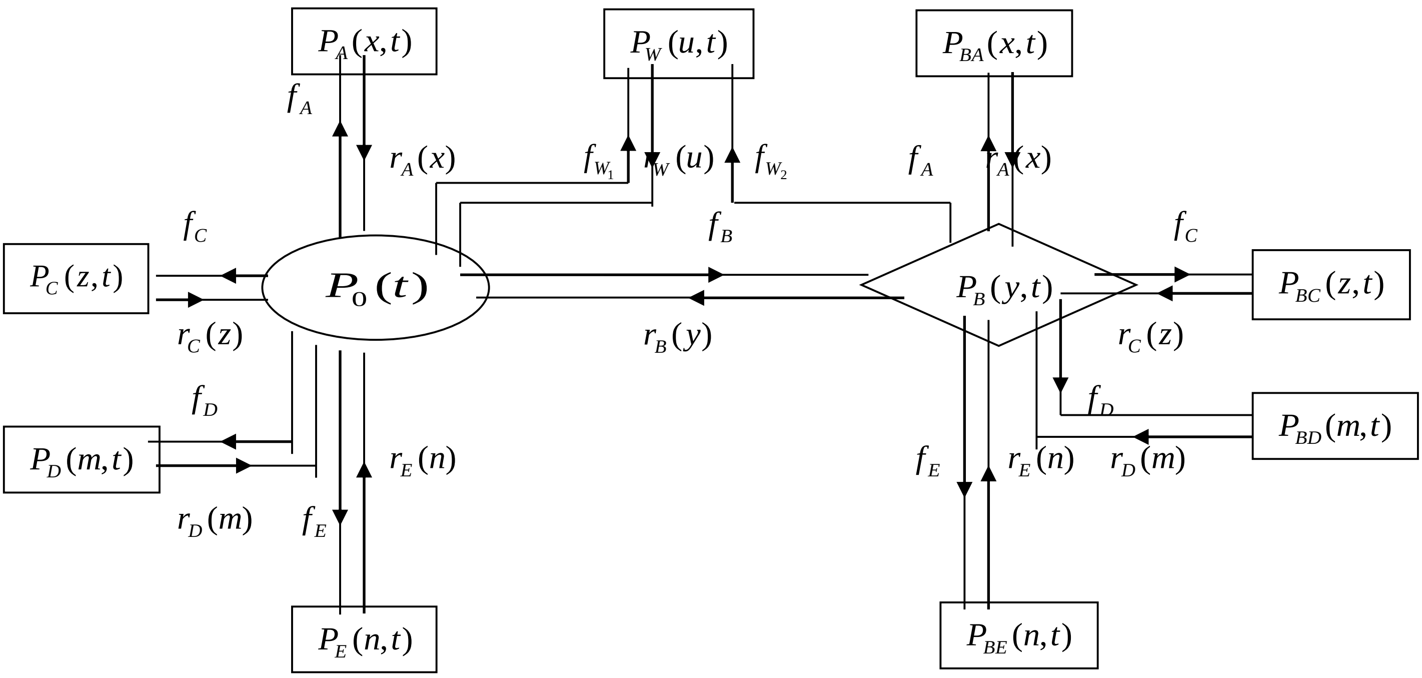


Fig-1(a): Summer Air conditioning system for hot and dry weather



States:

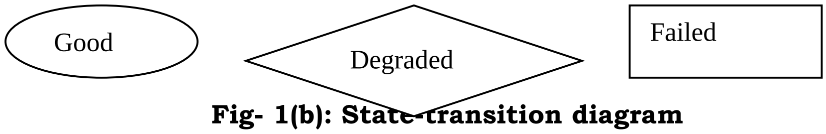


Fig- 1(b): State-transition diagram

3. NOTATIONS USED

The following notations have been used throughout in this model:

f_i	:	Failure rates of subsystem *i*.
$r_i(j)\Delta$	:	First order probability that i^{th} failure can be repaired in the time interval $(j, j+\Delta)$ conditioned that it was not repaired upto time *j*.
$P_0(t)$	:	Pr {at time *t*, the whole system is operable}.
$P_i(j,t)\Delta$	:	Pr {at time *t*, system suffers with i^{th} failure}. Elapsed repair time lies in the interval $(j, j+\Delta)$.
$P_W(u,t)\Delta$	:	Pr {at time *t*, system is failed due to wear-out}. Elapsed repair time lies in the interval $(u, u+\Delta)$.
$P_{Bi}(j,t)\Delta$	:	Pr {at time *t*, system is failed due to i^{th} failure while the subsystem B has already failed}.Elapsed repair time for subsystem *i* lies in the intervals $(j, j+\Delta)$.
$S_i(j)$	:	$\mu_i(j)\exp\left\{-\int r_i(j)dj\right\}$, $\forall$ *i* and *j*.
$\overline{F}(s)$	:	Laplace transform of the function F(t).
$D_i(j)$	:	$1-\overline{S}_i(j)/j,\ \forall\ i$ and j.
$M.T.T.F$	:	Mean time to failure.

4. FORMULATION OF MATHEMATICAL MODEL

Using probability considerations and limiting procedure, we obtain the following set of difference-differential equations which is continuous in time, discrete in space and governing the behaviour of considered system:

$$\left[\frac{d}{dt}+f_A+f_B+f_C+f_D+f_E+f_{W_1}\right]P_0(t)=\int_0^\infty P_A(x,t)r_A(x)dx+\int_0^\infty P_B(y,t)r_B(y)dy$$
$$+\int_0^\infty P_C(z,t)r_C(z)dz+\int_0^\infty P_D(m,t)r_D(m)dm \qquad \ldots(1)$$
$$+\int_0^\infty P_E(n,t)r_E(n)dn+\int_0^\infty P_W(u,t)r_W(u)du$$

$$\left[\frac{\partial}{\partial j}+\frac{\partial}{\partial t}+r_i(j)\right]P_i(j,t)=0 \qquad \ldots(2)$$

where i = A,C,D,E and j = x,z,m,n respectively.

$$\left[\frac{\partial}{\partial u}+\frac{\partial}{\partial t}+r_W(u)\right]P_W(u,t)=0 \qquad \ldots(3)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+f_A+f_C+f_D+f_E+f_{W_2}+r_B(y)\right]P_B(y,t)=0 \qquad \ldots(4)$$

$$\left[\frac{\partial}{\partial j}+\frac{\partial}{\partial t}+r_i(j)\right]P_{Bi}(j,t)=0 \qquad \ldots(5)$$

where i = A,C,D,E and j = x,z,m,n respectively.

Boundary conditions are:

$$P_i(0,t) = f_i\ P_0(t), \qquad \text{where } i = \text{A,C,D and E} \qquad \ldots(6)$$

$$P_W(0,t) = f_{W_1}\ P_0(t) + f_{W_2}\ P_B(t) \qquad \ldots(7)$$

$$P_B(0,t) = \int_0^\infty P_{BA}(x,t)r_A(x)dx + \int_0^\infty P_{BC}(z,t)r_C(z)dz + \int_0^\infty P_{BD}(m,t)r_D(m)dm + \int_0^\infty P_{BE}(n,t)r_E(n)dn + f_B P_0(t) \qquad \ldots(8)$$

$$P_{Bi}(0,t) = f_i\ P_B(t) \qquad \text{where } i = \text{A,C,D and E} \qquad \ldots(9)$$

Initial conditions are: …(10)

$$P_0(0) = 1, \text{ otherwise zero}$$

5. SOLUTION OF THE MODEL

In order to solve the above set of equations to obtain different state probabilities, taking Laplace transforms of equations (1) through (9) subjected to initial conditions (10), we get:

$$\left[s + f_A + f_B + f_C + f_D + f_E + f_{W_1}\right]\overline{P}_0(s) = 1 + \int_0^\infty \overline{P}_A(x,s)r_A(x)dx + \int_0^\infty \overline{P}_B(y,s)r_B(y)dy + \int_0^\infty \overline{P}_C(z,s)r_C(z)dz + \int_0^\infty \overline{P}_D(m,s)r_D(m)dm + \int_0^\infty \overline{P}_E(n,s)r_E(n)dn + \int_0^\infty \overline{P}_W(u,s)r_W(u)du \qquad \ldots(11)$$

$$\left[\frac{\partial}{\partial j} + s + r_i(j)\right]\overline{P}_i(j,s) = 0 \qquad \ldots(12)$$

where i = A,C,D,E and j = x,z,m,n respectively.

$$\left[\frac{\partial}{\partial u} + s + r_W(u)\right]\overline{P}_W(u,s) = 0 \qquad \ldots(13)$$

$$\left[\frac{\partial}{\partial y} + s + f_A + f_C + f_D + f_E + f_{W_2} + r_B(y)\right]\overline{P}_B(y,s) = 0 \qquad \ldots(14)$$

$$\left[\frac{\partial}{\partial j} + s + r_i(j)\right]\overline{P}_{Bi}(j,s) = 0 \qquad \ldots(15)$$

where i = A,C,D,E and j = x,z,m,n respectively.

$$\overline{P}_i(0,s) = f_i\ \overline{P}_0(s), \qquad \text{where } i = \text{A,C,D and E} \qquad \ldots(16)$$

$$\overline{P}_W(0,s) = f_{W_1}\ \overline{P}_0(s) + f_{W_2}\ \overline{P}_B(s) \qquad \ldots(17)$$

$$\overline{P}_B(0,s) = f_B\overline{P}_0(s) + \int_0^\infty \overline{P}_{BA}(x,s)r_A(x)dx + \int_0^\infty \overline{P}_{BC}(z,s)r_C(z)dz + \int_0^\infty \overline{P}_{BD}(m,s)r_D(m)dm + \int_0^\infty \overline{P}_{BE}(n,s)r_E(n)dn \qquad \ldots(18)$$

$$\overline{P}_{Bi}(0,s) = f_i\ \overline{P}_B(s) \qquad \text{where } i = \text{A,C,D and E} \qquad \ldots(19)$$

Now integrate equation (12) by using boundary conditions (16), we have

$$\overline{P}_i(j,s) = f_i\ \overline{P}_0(s)\exp\left\{-sj - \int r_i(j)dj\right\}$$

integrating this again w.r.t. 'j' from 0 to ∞, we get

$$\overline{P}_i(s) = f_i\,\overline{P}_0(s)\frac{1-\overline{S}_i(s)}{s} \quad \text{…(20)}$$

or, $\overline{P}_i(s) = f_i\,\overline{P}_0(s)D_i(s)$ for i = A,C,D and E

Similarly, equation (13) gives on integration subjected to boundary condition (17):

$$\overline{P}_W(s) = \left[f_{W_1}\,\overline{P}_0(s) + f_{W_2}\,\overline{P}_B(s)\right]D_W(s) \quad \text{…(21)}$$

Integrate (15) by making use of (19), we obtain

$$\overline{P}_{Bi}(j,s) = f_i\,\overline{P}_B(s)\exp\left\{-sj - \int r_i(j)dj\right\}$$

integrating this again w.r.t. j from 0 to ∞, we have

$$\overline{P}_{Bi}(s) = f_i\,\overline{P}_B(s)D_i(s) \quad \text{for } i = \text{A,C,D and E} \quad \text{…(22)}$$

Now simplifying (18) subjected to relevant relations, we get

$$\overline{P}_B(0,s) = f_B\,\overline{P}_0(s) + f_A\,\overline{P}_B(s)\overline{S}_A(s) + f_C\,\overline{P}_B(s)\overline{S}_C(s) + f_D\,\overline{P}_B(s)\overline{S}_D(s) + f_E\,\overline{P}_B(s)\overline{S}_E(s) \quad \text{…(23)}$$

Equation (14) gives on integration:

$$\overline{P}_B(y,s) = \overline{P}_B(0,s)\exp\left\{-\left(s + f_A + f_C + f_D + f_E + f_{W_2}\right)y - \int r_B(y)dy\right\}$$

integrating this again w.r.t. y from 0 to ∞, we obtain

$$\overline{P}_B(s) = \overline{P}_B(0,s)D_B\left(s + f_A + f_C + f_D + f_E + f_{W_2}\right)$$

or, $\overline{P}_B(s) = \overline{P}_B(0,s)D_B(N)$

where, $N = s + f_A + f_C + f_D + f_E + f_{W_2}$

using (23), it gives

$$\overline{P}_B(s)\left[1 - \left\{f_A\,\overline{S}_A(s) + f_C\,\overline{S}_C(s) + f_D\,\overline{S}_D(s) + f_E\,\overline{S}_E(s)\right\}D_B(N)\right] = f_B\,\overline{P}_0(s)D_B(N)$$

$$\therefore \overline{P}_B(s) = \frac{f_B D_B(N)\overline{P}_0(s)}{1 - \left\{f_A\,\overline{S}_A(s) + f_C\,\overline{S}_C(s) + f_D\,\overline{S}_D(s) + f_E\,\overline{S}_E(s)\right\}D_B(N)} \quad \text{…(24)}$$

or, $\overline{P}_B(s) = A(s)\overline{P}_0(s)$ (Say)

Finally, simplifying equation (11) with the help of relevant expressions, we have

$$\overline{P}_0(s) = \frac{1}{C(s)}$$

Thus, we obtain the following L.T. of various state probabilities, depicted in fig-1(b), in terms of C(s):

$$\overline{P}_0(s) = \frac{1}{C(s)} \quad \text{…(25)}$$

$$\overline{P}_i(s) = \frac{f_i\,D_i(s)}{C(s)}, \quad i = \text{A,C,D and E} \quad \text{…(26)}$$

$$\overline{P}_W(s) = \frac{1}{C(s)}\left[f_{W_1} + f_{W_2}\,A(s)\right]D_W(s) \quad \text{…(27)}$$

$$\overline{P}_B(s) = \frac{A(s)}{C(s)} \quad \text{…(28)}$$

$$\overline{P}_{Bi}(s) = \frac{f_i\,A(s)D_i(s)}{C(s)}, \quad i = \text{A,C,D and E} \quad \text{…(29)}$$

where, $A(s) = \dfrac{f_B D_B(N)}{1 - \left\{f_A\,\overline{S}_A(s) + f_C\,\overline{S}_C(s) + f_D\,\overline{S}_D(s) + f_E\,\overline{S}_E(s)\right\}D_B(N)}$ …(30)

$N = s + f_A + f_C + f_D + f_E + f_{W_2}$ …(31)

and $C(s) = s + f_A + f_B + f_C + f_D + f_E + f_{W_1} - f_A\,\overline{S}_A(s) - f_C\,\overline{S}_C(s) - f_D\,\overline{S}_D(s)$

$$- f_E \overline{S}_E(s) - \left[f_{W_1} + f_{W_2} A(s)\right]\overline{S}_W(s)$$
$$- \left[f_B + \left\{f_A \overline{S}_A(s) + f_C \overline{S}_C(s) + f_D \overline{S}_D(s) + f_E \overline{S}_E(s)\right\} A(s)\right]\overline{S}_B(N) \quad \ldots(32)$$

VERIFICATION

It is interesting to note here that

sum of equations (25) through (29) $= \dfrac{1}{s}$ …(33)

6. STEADY-STATE BEHAVIOUR OF THE SYSTEM

Using final value theorem in L.T., *viz*, $\lim_{t\to\infty} P(t) = \lim_{s\to 0} s\overline{P}(s) = P(say)$, provided the limit on LHS exists, in equations (25) through (29), we obtain the following steady-state behaviour of the considered system:

$$P_0 = \frac{1}{C'(0)} \quad \ldots(34)$$

$$P_i = \frac{f_i M_i}{C'(0)} \quad , \quad i = \text{A,C,D and E} \quad \ldots(35)$$

$$P_W = \frac{1}{C'(0)}\left[f_{W_1} + f_{W_2} A(0)\right]M_W \quad \ldots(36)$$

$$P_B = \frac{A(0)}{C'(0)} \quad \ldots(37)$$

$$P_{Bi} = \frac{f_i A(0) M_i}{C'(0)} \quad , \quad i = \text{A,C,D and E} \quad \ldots(38)$$

where, $C'(0) = \left[\dfrac{d}{ds} C(s)\right]_{s=0}$

$M_i = -\overline{S}_i{}'(0) =$ mean time to repair i^{th} failure.

and $A(0) = \dfrac{f_B D_B(N-s)}{1-(f_A + f_C + f_D + f_E) D_B(N-s)}$

7. PARTICULAR CASE

When repairs follow exponential time distribution

In this case, setting $\overline{S}_i(j) = \dfrac{r_i}{j + r_i}, \forall\ i \text{ and } j$ in equations (25) through (29) we obtain the following L.T. of various states probabilities of fig-1(b):

$$\overline{P}_0(s) = \frac{1}{E(s)} \quad \ldots(39)$$

$$\overline{P}_i(s) = \frac{f_i}{E(s)(s + r_i)} \quad , \quad i = \text{A,C,D and E} \quad \ldots(40)$$

$$\overline{P}_W(s) = \frac{1}{E(s)}\left[f_{W_1} + f_{W_2} Q(s)\right]\frac{1}{s + r_W} \quad \ldots(41)$$

$$\overline{P}_B(s) = \frac{Q(s)}{E(s)} \quad \ldots(42)$$

$$\overline{P}_{Bi}(s) = \frac{f_i Q(s)}{E(s)(s + r_i)} \quad , \quad i = \text{A,C,D and E} \quad \ldots(43)$$

where, $$Q(s)=\frac{f_B\left[1-\bar{S}_B(N)\right]}{N-\left(\frac{f_A r_A}{s+r_A}+\frac{f_C r_C}{s+r_C}+\frac{f_D r_D}{s+r_D}+\frac{f_E r_E}{s+r_E}\right)\left[1-\bar{S}_B(N)\right]} \quad \ldots(44)$$

and $$E(s)=s+f_A+f_B+f_C+f_D+f_E+f_{W_1}-\frac{f_A r_A}{s+r_A}-\frac{f_C r_C}{s+r_C}-\frac{f_D r_D}{s+r_D}-\frac{f_E r_E}{s+r_E}$$

$$-\left[f_{W_1}+f_{W_2}Q(s)\right]\frac{r_W}{s+r_W}$$

$$-\left[f_B+\left\{\frac{f_A r_A}{s+r_A}+\frac{f_C r_C}{s+r_C}+\frac{f_D r_D}{s+r_D}+\frac{f_E r_E}{s+r_E}\right\}Q(s)\right]\frac{r_B}{N+r_B} \quad \ldots(45)$$

N has been mentioned earlier in equation (31).

8. RELIABILITY AND M.T.T.F. OF THE SYSTEM

We have from equation (25)

$$\bar{R}(s)=\frac{1}{s+f_A+f_B+f_C+f_D+f_E+f_{W_1}}$$

Taking inverse L.T., we get

$$R(t)=\exp\left\{-\left(f_A+f_B+f_C+f_D+f_E+f_{W_1}\right)t\right\} \quad \ldots(46)$$

Also, $$M.T.T.F.=\int_0^\infty R(t)dt$$

$$=\frac{1}{f_A+f_B+f_C+f_D+f_E+f_{W_1}} \quad \ldots(47)$$

9. AVAILABILITY OF THE SYSTEM

From equations (25) and (28), we obtain

$$\bar{P}_{up}(s)=\frac{1}{s+f_A+f_B+f_C+f_D+f_E+f_{W_1}}\left[1+\frac{f_B}{s+f_A+f_B+f_C+f_D+f_E+f_{W_2}}\right]$$

Taking inverse L.T., we get

$$P_{up}(t)=\left(1+\frac{f_B}{f_{W_2}-f_B-f_{W_1}}\right)\exp\left\{-\left(f_A+f_B+f_C+f_D+f_E+f_{W_1}\right)t\right\}$$

$$-\frac{f_B}{f_{W_2}-f_B-f_{W_1}}\exp\left\{-\left(f_A+f_C+f_D+f_E+f_{W_2}\right)t\right\} \quad \ldots(48)$$

Again, $$P_{down}(t)=1-P_{up}(t) \quad \ldots(49)$$

10. NUMERICAL COMPUTATON

For a numerical computation, let us consider the following values:

$f_A=0.001$, $f_B=0.002$, $f_C=0.06$, $f_D=0.04$, $f_E=0.008$, $f_{W_1}=0.003$, $f_{W_2}=0.009$ and $t=0,1,2,--$.

Using these values in equations (46), (47) and (48) we obtain the table-1, 2 and 3, respectively. The corresponding graphs have been shown in fig-2, 3 and 4 respectively.

11. RESULTS AND DISCUSSION:

Table-1 gives the values of reliability of considered system for various values of time *t*. Its graph has been shown in fig-2. Analysis of table-1 and fig-2 reveal that the reliability of considered system decreases approximately in constant manner and there are no sudden jumps in the values of reliability.

Table-2 gives the values of availability of considered system for different values of time *t*. Its graph has been shown in fig-3. Critical examination of table-2 and fig-3 yield that value of availability decreases rapidly in the beginning but thereafter it decreases constantly.
Table-3 gives the values of M.T.T.F. of considered system for different values of failure rate of subsystem *B*. Its graph has been shown in fig-4. Analysis of table-3 and fig-4 yield that value of M.T.T.F. decreases catastrophically.

t	R(t)
0	1
1	0.892258
2	0.796124
3	0.710348
4	0.633814
5	0.565525
6	0.504595
7	0.450229
8	0.40172
9	0.358438
10	0.319819

Table-1: Reliability

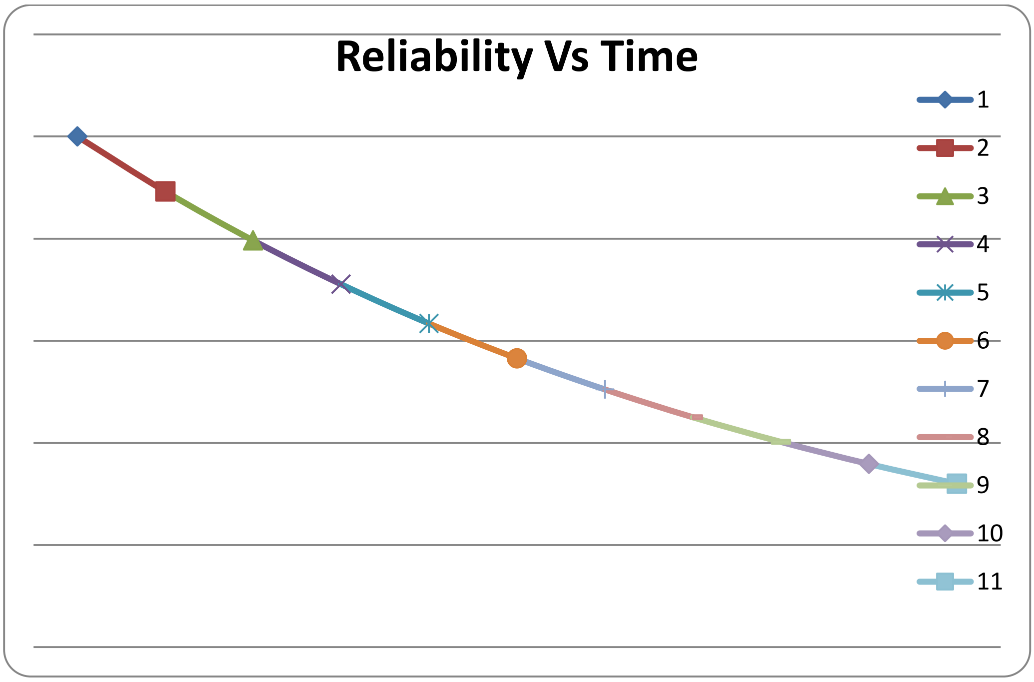


Fig-2: Time Vs Reliability

t	$P_{up}(t)$
0	1
1	0.894039
2	0.799296
3	0.714585
4	0.638844
5	0.571125
6	0.510578
7	0.456444
8	0.408046
9	0.364775
10	0.326089

Table-2

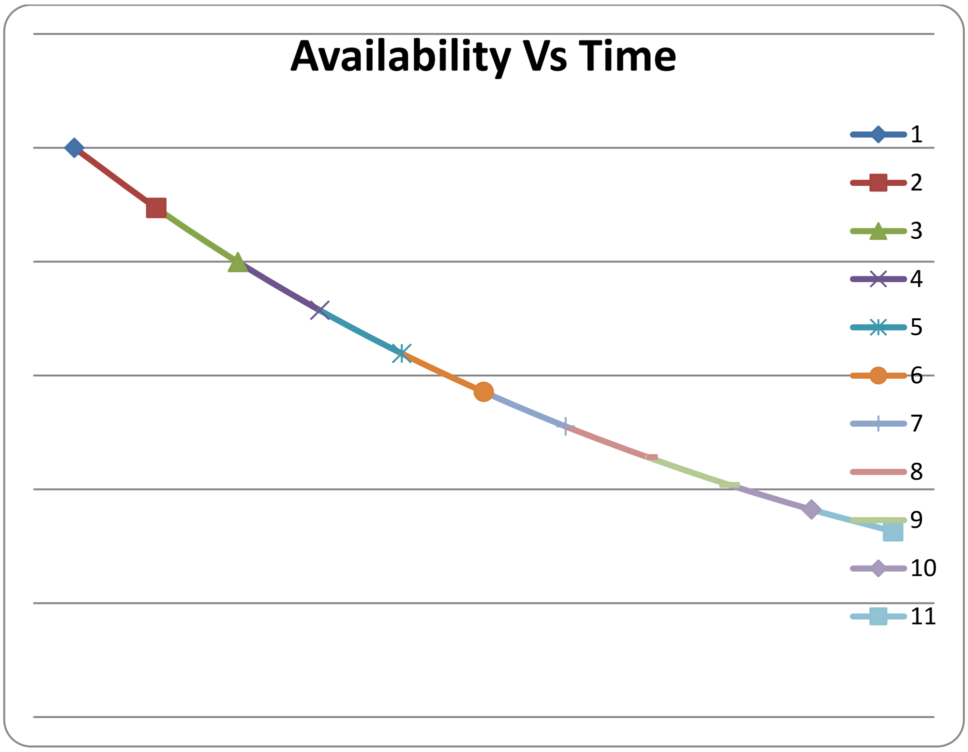


Fig-3: Time Vs Availability

f_B	M.T.T.F.
0	8.928571
0.001	8.849558
0.002	8.77193
0.003	8.695652
0.004	8.62069
0.005	8.547009
0.006	8.474576
0.007	8.403361
0.008	8.333333
0.009	8.264463
0.01	8.196721

Table-3

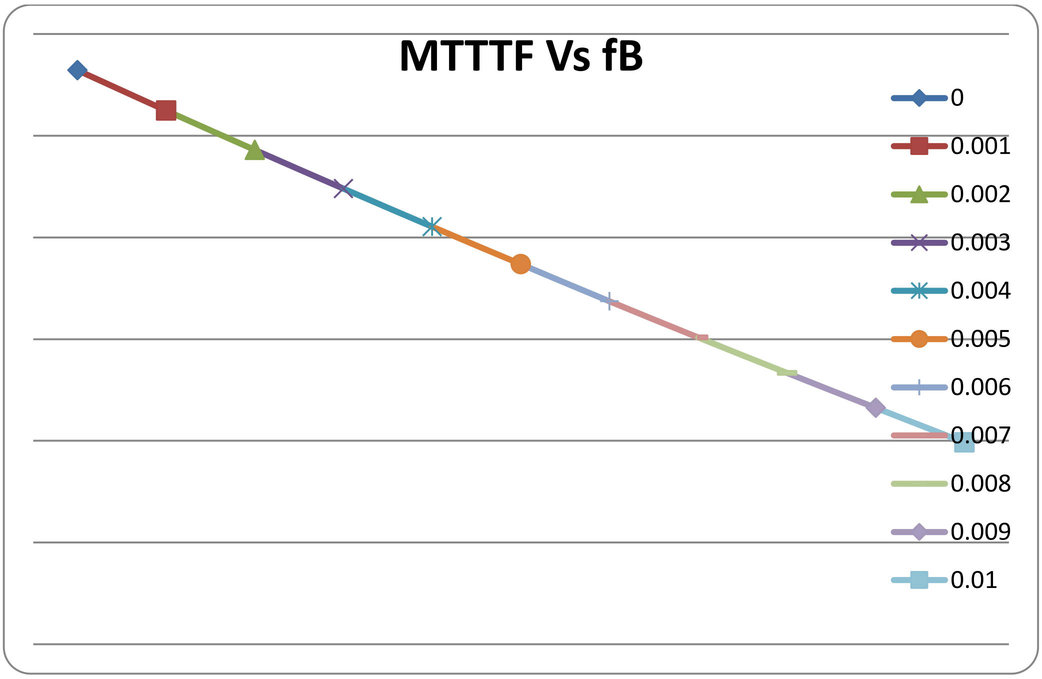


Fig-4: F_B Vs MTTF

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